Fermi National Accelerator Laboratory

The Charmonium Spectrum on the Lattice:
A Status Report*

Aida X. El-Khadra

Fermi National Accelerator Laboratory

P.O. Box 500, Batavia, IL 60510

Abstract

We present our most recent results on the Charmonium spectrum using relativistic Wilson fermions. We study the dependence of the spectrum on the charm quark mass and the Wohlert-Sheikholeslami improvement term.

---

*Talk given at *Lattice '92*, Amsterdam, September 15-19, 1992

The Charmonium Spectrum on the Lattice: A Status Report*

Aida X. El-Khadra

Theory Group, Fermilab, P.O. Box 500, Batavia Il 60510

We present our most recent results on the charmonium spectrum using relativistic Wilson fermions. We study the dependence of the spectrum on the charm quark mass and the Wohlert-Sheikholeslami improvement term.

1. INTRODUCTION

This is a status report on our calculation of the charmonium spectrum. Our first results were presented last year [1]. An important aspect of this calculation, the determination of α_s from the charmonium spectrum, has already been reported separately [2]. The detailed results of this study will be presented in a future publication [3].

The calculation of experimentally well measured quantities in the charmonium spectrum serves several purposes. The investigation of physical quantities that are sensitive to the systematic errors present in lattice calculations can lead to a better understanding and possibly the removal of these errors. On the other hand, a quantity that is relatively independent of the systematic errors of the lattice calculation can be used to obtain information about fundamental parameters of the Standard Model.

In Refs. [1, 2] we argued that the 1P-1S splitting falls into the latter class of quantities and therefore lends itself to an accurate determination of the strong coupling constant of QCD. The splitting between the J/ψ and η_c, on the other hand, is expected to strongly depend on the leading lattice spacing errors. This splitting and the leptonic decay amplitude of the J/ψ are expected to also show some dependence on the charm quark mass. One might therefore use these two quantities in a study of the former type.

Some of the details of the calculation are described in section 2. The effect of the omission of sea quarks has already been discussed in detail in our analysis of α_s. In section 3 we discuss their effect on the hyperfine splitting and the leptonic decay amplitude of the J/ψ. The results of our study of the dependence of the spectrum on the

charm quark mass and $\mathcal{O}(a)$ lattice spacing errors are presented in section 4.

2. THE CALCULATION

We analyze three different lattices ($12^3 \times 24$, $16^3 \times 32$, 24^4) at three different couplings ($\beta = 5.7, 5.9, 6.1$) such that the volumes are similar while the lattice spacing varies by about a factor of two (see Ref. [1, 2] for more details). In order to investigate the effects of lattice spacing errors on the spectrum, we use Wilson fermions with ($c = 1.4$) and without ($c = 0$) the $\mathcal{O}(a)$ correction term [4]. We study the charm quark mass dependence by varying the charm hopping parameter, κ_{charm}, over a reasonable range for each lattice and fermion action (or choice of c).

The 2-pt. functions used for the mass splittings have already been described in detail in Ref. [1]. The mesons are created and destroyed by operators, $\chi(x) = \sum_r \psi(x + r)\Gamma\bar{\psi}(x)S(r)$, with exponential spreading functions, $S(r)$, that optimize the overlap with the ground states. The leptonic matrix element for the J/ψ is extracted from the 2-pt. function ($\Gamma = \gamma_i$)

$$G_2^V(t; \mu, i) = \sum_x \langle V_\mu(x)\chi_i^\dagger(0)\rangle \ . \tag{1}$$

$V_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ is the local vector current. With the convention $\langle p|p'\rangle = (2\pi)^3\delta^3(p - p')$, Eq. (1) takes the asymptotic form

$$G_2^V(t; \mu, i) \rightarrow \langle 0|V_\mu|J/\psi\rangle\langle J/\psi|\chi^\dagger|0\rangle e^{-mt} \ . \tag{2}$$

We parametrize the matrix element in terms of V_ψ as

$$\langle 0|V_\mu|J/\psi\rangle = \epsilon_\mu V_\psi \ . \tag{3}$$

V_ψ is related to the decay constant f_ψ^{-1} used by other groups [5] by $V_\psi = (m_\psi^3/2)^{1/2}f_\psi^{-1}$. We

renormalize the current non-perturbatively with the charge matrix element $\langle J/\psi|V_4|J/\psi\rangle = Z_V^{-1}$. As remarked in Refs. [6, 7], the perturbative calculation of the local current renormalization Z_V (using improved perturbation theory [8]) reproduces the non-perturbative result to a few % in the case of Wilson ($c = 0$) fermions, if the quark fields are properly normalized.

3. THE EFFECT OF SEA QUARKS

A nonrelativistic system, like charmonium, is well described by a potential. We can therefore understand the dominant effects of the sea quarks on the spectrum via their effect on the potential. Following the arguments of Ref. [2], in setting the scale a^{-1} with the 1P-1S splitting, we require in effect the potentials of the full and the quenched theory to match at the relevant physics scale which is the intermediate distance scale of $\sim 400 - 750$ MeV. We then expect the (perturbative) short distance potential to be too weak, as a result of the zero flavor β-function being slightly too large.

For quantities that are dominated by short distance effects, like the hyperfine splitting and leptonic decay amplitude of the J/ψ, we have to consider the sea quark effects on the short distance potential. This is best investigated within the context of a potential model. One expects for the hyperfine splitting to lowest order [9]

$$\Delta m(J/\psi - \eta_c) \sim \frac{\alpha_s(m_c)}{m_c^2}|\Psi(0)|^2 \ . \tag{4}$$

Similarly, according to the van Royen - Weisskopf formula we expect for the leptonic matrix element

$$V_\psi \sim |\Psi(0)| \ . \tag{5}$$

Thus, the effect of the sea quarks on the hyperfine splitting and the leptonic matrix element can be estimated by their effect on the wave function at the origin, $\Psi(0)$, in a potential model.

The Richardson potential is convenient for our purposes, because it incorporates asymptotic freedom and confinement in a simple form [10]:

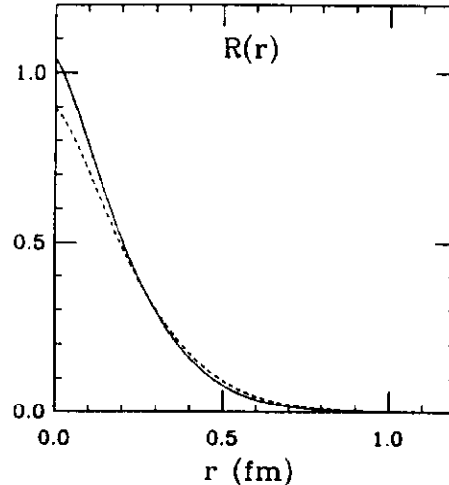$$V(q^2) = C_F \frac{4\pi}{\beta_0^{(n_f)}} \frac{1}{q^2 \ln\left(1 + q^2/\Lambda^2\right)} \ , \tag{6}$$

Figure 1. The radial wave function for the 1S state in the Richardson potential model. The solid line is for $n_f = 3$ and the dashed line for $n_f = 0$.

with $\beta_0^{(n_f)} = 11 - 2n_f/3$. Figure 1 shows the wave functions of the 1S state, obtained from fitting the Richardson potential to the experimental charmonium spectrum with $n_f = 3$ and $n_f = 0$ in comparison. We find for the ratio of wave functions at the origin:

$$\frac{\Psi^{(0)}(0)}{\Psi^{(3)}(0)} = 0.86 \ . \tag{7}$$

The hyperfine splitting receives an additional correction from $\alpha_s(m_c)$. The calculation of the effects of the sea quarks on $\alpha_s(\pi/a)$ is described in detail in Ref. [2]. The resulting correction for $\alpha_s(m_c)$ is analogously:

$$\frac{\alpha_s^{(0)}(m_c)}{\alpha_s^{(3)}(m_c)} = 0.81 \pm 0.06 \ . \tag{8}$$

In summary, we estimate the quenched hyperfine splitting to be reduced by 40 % from the experimental value, $\Delta m(J/\psi - \eta_c)^{\mathrm{exp}} = 117.3$ MeV to

$$\Delta m(J/\psi - \eta_c)^{\mathrm{quenched}} \simeq 70 \text{ MeV} \ . \tag{9}$$

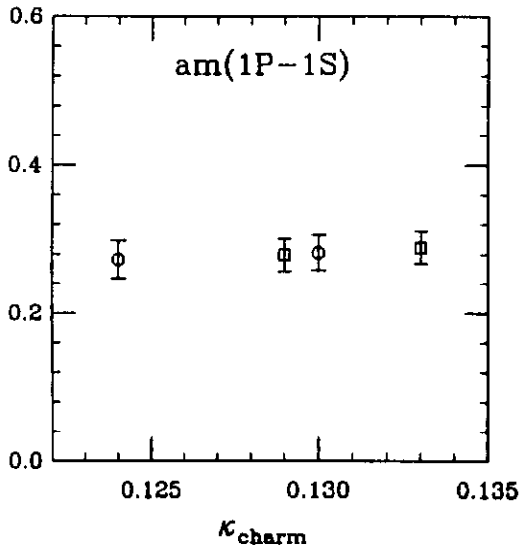The quenched leptonic matrix element just recieves the correction in Eq. (7), a 14 % reduc-

2

Figure 2. The 1P-1S splitting in lattice units vs. κ_{charm} on the $16^3 \times 32$, $\beta = 5.9$ lattice; the circles are for $c = 1.4$, the squares are for $c = 0$.

tion from the experimental result $V_\psi^{exp} = 0.509$ GeV$^{3/2}$:

$$V_\psi^{quenched} = 0.438\,\text{GeV}^{3/2} \ . \tag{10}$$

There is certainly no reason to believe that the sea quarks do not change the potential (nonperturbatively) in a way that has not been taken into account here. Eqs. (9) and (10) are therefore only rough estimates of the possible effect. However, from the above considerations it should be expected that the omission of sea quarks reduces both the hyperfine splitting and the leptonic width from their experimental values.

4. RESULTS

Figures 2, 3 and 4 show the 1P-1S splitting, the J/ψ - η_c splitting, and the leptonic matrix element of the J/ψ as functions of the charm hopping parameter for Wilson fermions with ($c = 1.4$) and without ($c = 0$) the improvement term on the $16^3 \times 32$ ($\beta = 5.9$) lattice as representative examples of our results on all three lattices. As expected, we find the dependence of the 1P-1S
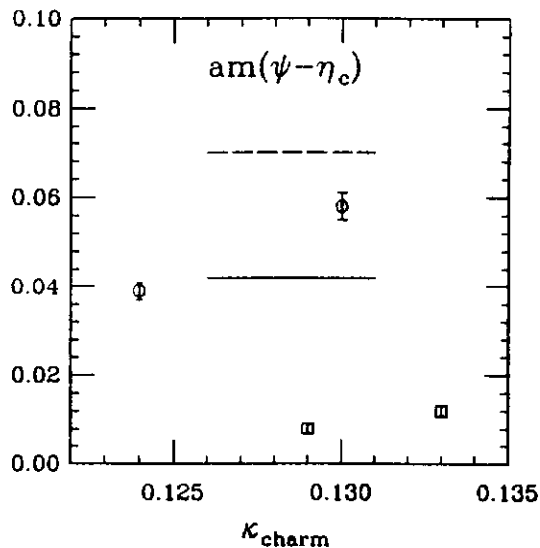

Figure 3. The J/ψ-η_c splitting in lattice units vs. κ_{charm} on the $16^3 \times 32$, $\beta = 5.9$ lattice; the circles are for $c = 1.4$, the squares are for $c = 0$. The dashed line is the experimnetal splitting; the solid line is the splitting of Eq. (9).
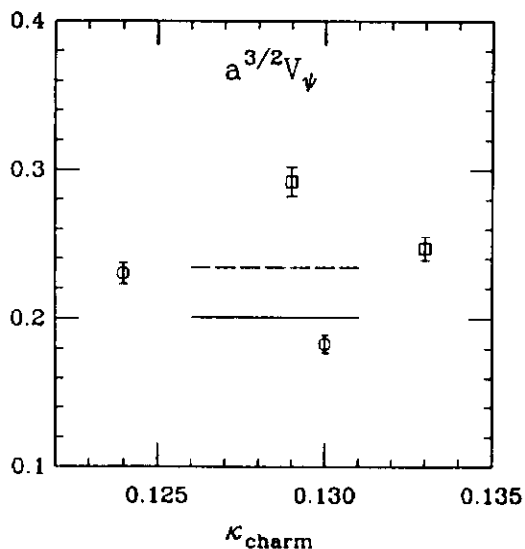

Figure 4. The leptonic matrix element, V_ψ in lattice units. vs. κ_{charm} on the $16^3 \times 32$. $\beta = 5.9$ lattice; the circles are for $c = 1.4$, the squares are for $c = 0$. The dashed line is the experimental value; the solid line is the value of Eq. (10).
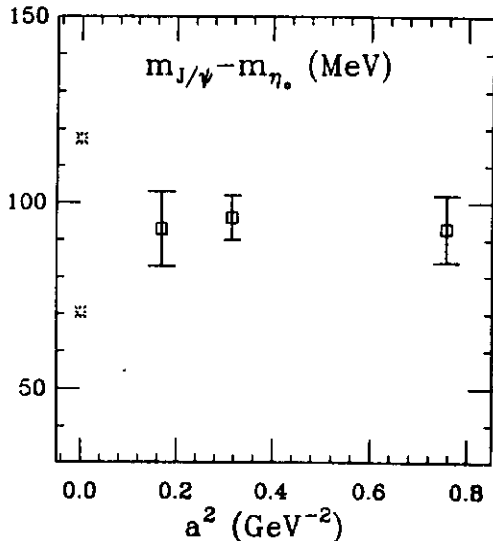
3

Figure 5. The J/ψ-η_c splitting vs. a^2. The two points on the left (burst) represent the experimental splitting (upper) and the expectation for the quenched result (lower).

ting in physical units is graphed as a function of a^2 for better visiblility. Also shown are the experimental and the expected quenched values. We find very little variation of the hyperfine splitting with the lattice spacing, indicating small residual lattice spacing errors. Our final result

$$\Delta m(J/\psi - \eta_c) = 93 \pm 10 \text{ MeV} \qquad (11)$$

(statistical error only) lies within the expectations for a quenched calculation, slightly below the experimentally measured splitting.

ACKNOWLEDGEMENTS

splitting on the charm quark mass (parametrized by κ_{charm}) and the coefficient of the improvement term, c, to be very weak, smaller than our statistical errrors. This confirms our previous arguments [1, 2] to use this splitting to determine the scale a^{-1} in lattice calculations and subsequently extract the strong coupling constant from it. Our previous result for α_s remains unchanged after this study.

Both the hyperfine splitting as well as the leptonic matrix element vary significantly with the two action parameters, κ_{charm} and c. However, from the considerations in section 3, it is clear that a phenomenological determination of these parameters is not possible within the quenched approximation without a better understanding of the effects of the sea quarks on the static potential. We therefore take the mean field value $c = 1.4$ as our best estimate for the coefficient of the improvement term. Setting the charm quark mass with the leptonic matrix element, V_ψ (using Eq. (10)), we extract hyperfine splittings as shown in figure 5 for all three lattices. The split-

REFERENCES

1   P. B. Mackenzie, Nucl. Phys. B (Proc. Suppl.) 26 (1992) 369; A. X. El-Khadra, Nucl. Phys. B (Proc. Suppl.) 26 (1992) 372.
2   A. X. El-Khadra, G. M. Hockney, A. S. Kronfeld, P. B. Mackenzie, Phys. Rev. Lett. 69 (1992) 729.
3   A. X. El-Khadra, G. M. Hockney, A. S. Kronfeld, P. B. Mackenzie, manuscript in preparation.
4   B. Sheikholeslami and R. Wohlert, Nucl. Phys. B 259 (1985) 572.
5   C. R. Allton et al. (The UKQCD collaboration), Southampton Preprint SHEP 91/92-27.
6   A. S. Kronfeld, these proceedings.
7   P. B. Mackenzie, these proceedings.
8   G. P. Lepage, P. B. Mackenzie, Fermilab preprint PUB-91/355-T.
9   For a review on heavy quarkonia see for example: W. Kwong, J. L. Rosner, C. Quigg, Ann. Rev. Nucl. Part. Sci. 37 (1987) 325.
10 J. L. Richardson, Phys. Lett. 82B (1979) 272.

4